In the triangles FCB and FBD, $\angle FCB=\angle FBA$, since arc AF=arc FB; also $\angle CFB$ is common, hence the triangles are similar, and $FC:FB=FB:FD$; but $FL(=FC):FB=FB:FH$. Therefore $FH=FD$ and $HL=CD$.

Hence in the triangle ABC, AB is the given base, $\angle ACB$ the given vertical angle, and CD the given bisector, and the triangle is satisfied in every condition.

Also solved by L. E. Newcomb, and A. H. Holmes.

CALCULUS.

233. Proposed by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Stroud, England.

Prove that $\int_0^\infty \frac{x^{a-1}dx}{1+2x\cos\theta+x^2}=\frac{\pi\sin(1-a)\theta}{\sin a\pi\sin\theta}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\int_0^\infty \frac{a^{-1}dx}{1+2x\cos\theta+x^2}=\frac{1}{a}\int_0^\infty \frac{dx}{\sin^2\theta+(x+\cos\theta)^2}$$

$$=\frac{1}{a\sin\theta}\tan^{-1}\left(\frac{x+\cos\theta}{\sin\theta}\right)_0^\infty=\frac{\theta}{a\sin\theta}$$

The problem giving the result stated is as follows:

$$\int_0^\infty \frac{x^{a-1}dx}{1+2x\cos\theta+x^2}=\frac{1-\sqrt{(-1)}\cot\theta}{2}\int_0^\infty \frac{x^{a-2}dx}{x+\cos\theta+\sqrt{(-1)}\sin\theta}$$

$$+\frac{1+\sqrt{(-1)}\cot\theta}{2}\int_0^\infty \frac{x^{a-2}dx}{x+\cos\theta-\sqrt{(-1)}\sin\theta}=P.$$

Let $x=y[\cos\theta\pm\sqrt{(-1)}\sin\theta]$, the plus sign for the first term, the minus sign for the second term.

$$\therefore P=\tfrac{1}{2}[1-\sqrt{(-1)}\cot\theta]\,[\cos(a-2)\theta+\sqrt{(--1)}\sin(a-2)\theta]\int_0^\infty \frac{y^{a-2}dy}{1+y}$$

$$+\tfrac{1}{2}[1+\sqrt{(-1)}\cot\theta]\,[\cos(a-2)\theta-\sqrt{(-1)}\sin(a-2)\theta]\int_0^\infty \frac{y^{a-2}dy}{1+y}.$$

$$\therefore P=[\cos(a-2)\theta+\sin(a-2)\theta\cot]\int_0^\infty \frac{y^{a-2}dy}{1+y}$$

$$=\frac{\sin(a-1)\theta}{\sin\theta}\int_0^\infty \frac{y^{a-2}dy}{1+y}=\frac{\sin(a-1)\theta}{\sin\theta}\cdot\frac{\pi}{\sin(a-1)\pi}.$$

$$\therefore P=\frac{\pi\sin(1-a)\theta}{\sin\theta\sin a\pi}$$

This problem was incorrectly stated, the error being due to an oversight in reading proof. It is correctly stated above, the numerator being x^{a-1} instead of a^{-1}. ED. F.